

Holographic Superconductors with Power-Maxwell field

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ABSTRACT: With the Sturm-Liouville analytical and numerical methods, we investigate the behaviors of the holographic superconductors by introducing a complex charged scalar field coupled with a Power-Maxwell field in the background of d -dimensional Schwarzschild AdS black hole. We note that the Power-Maxwell field takes the special asymptotical solution near boundary which is different from all known cases. We find that the larger power parameter q for the Power-Maxwell field makes it harder for the scalar hair to be condensated. We also find that, for different q , the critical exponent of the system is still $1/2$, which seems to be an universal property for various nonlinear electrodynamics if the scalar field takes the form of this paper.

KEYWORDS: Holographic superconductors, Power-Maxwell field, AdS black hole.

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1. Introduction

The AdS/CFT correspondence [1, 2, 3] indicates that a weak coupling gravity theory in a d -dimensional anti-de Sitter spacetime can be related to a strong coupling conformal field theory on the $(d - 1)$ -dimensional boundary. Gubser first [4, 5] suggested that near the horizon of a charged black hole there is in operation a geometrical mechanism parameterized by a charged scalar field of breaking a local $U(1)$ gauge symmetry. Then, the gravitational dual of the transition from normal to superconducting states in the boundary theory was constructed. This dual consists of a system with a black hole and a charged scalar field, in which the black hole admits scalar hair at temperature lower than a critical temperature, but does not possess scalar hair at higher temperatures [6]. Since the AdS/CFT duality is a valuable tool for investigating strongly coupled gauge theories, the application might offer new insight into the investigation of strongly interacting condensed matter systems where the perturbational methods are no longer available. Therefore, much attention has been given to the studies of the AdS/CFT duality to condensed matter physics and in particular to superconductivity [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26] recently.

Because the Maxwell theory is only a special case or a leading order in the expanded form of nonlinear electrodynamics, the nonlinear electrodynamics which carries more information than the Maxwell field has been a subject of research for

many years [27, 28, 29, 30, 31, 32]. Heisenberg and Euler [27] noted that quantum electrodynamics predicts that the electromagnetic field behaves nonlinearly through the presence of virtual charged particles. Now we list the main nonlinear electrodynamics as follows: (i) Born and Infeld [28] presented a classical nonlinear theory of electromagnetism which contains many symmetries common to the Maxwell theory despite its nonlinearity. The Lagrangian density for Born-Infeld theory is $\mathcal{L}_{BI} = 4b^2 \left(1 - \sqrt{1 + \frac{F^2}{2b^2}} \right)$ with $F^2 = F_{\mu\nu}F^{\mu\nu}$ and the coupling parameter b is related to the string tension α' as $b = 1/(2\pi\alpha')$. This Lagrangian reduces to the Maxwell case in the weak-coupling limit $b \rightarrow \infty$. (ii) The action of Power-Maxwell field [33, 34, 35, 30, 36] is taken as power-law function of the form $\mathcal{L}_{PM} = -\beta F^{2q}$, where β is a coupling constant and q is a power parameter. It is interesting that the conformal invariance $g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu}$, $A_\mu \rightarrow A_\mu$ is realized for the power parameter $q = d/4$ where d is the dimensions of the spacetime [33]. (iii) The nonlinear electromagnetic Lagrangian that contains logarithmic terms appears in the description of vacuum polarization effects. The term were obtained as exact 1-loop corrections for electrons in a uniform electromagnetic field background by Euler and Heisenberg [27]. A simple example of a Born-Infeld-like Lagrangian with a logarithmic term, that can be added as a correction to the original Born-Infeld one, was discussed in Ref. [37]. In an arbitrary dimension, the logarithmic electromagnetic lagrangian has the form $\mathcal{L}_{BI} = -b^2 \ln \left(1 + \frac{F^2}{b^2} \right)$ where b is a coupling constant. The Lagrangian tends to the Maxwell case in the weak-coupling limit $b \rightarrow \infty$.

It is well known that the properties of holographic superconductors depend on behavior of the electromagnetic field coupled with the charged scalar field in the system. Motivated by the recent studies and the fact that, within the framework of AdS/CFT correspondence, the different electromagnetic action is expected to modify the dynamics of the dual theory, in this paper we will investigate the behavior of the holographic superconductors with the Power-Maxwell field in the background of a d -dimensional Schwarzschild AdS black hole, and to see how the Power-Maxwell field affect the formation of the scalar hair and the critical exponent of the system.

The paper is organized as follows. In Sec. II, we give the holographic dual of d -dimensional Schwarzschild AdS black hole by introducing a complex charged scalar field coupled with a Power-Maxwell field. In Sec. III, we explore the relations between critical temperature and charge density. In Sec. IV, we study the critical exponents of the holographic superconductor model with the Power-Maxwell field. We summarize and discuss our conclusions in the last section.

2. Holographic dual of d -dimensional Schwarzschild AdS black hole

In order to study a superconductor dual to a AdS black hole configuration in the

probe limit, we consider the d -dimensional Schwarzschild AdS black hole

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 dx_i dx^i, \quad (2.1)$$

with

$$f(r) = r^2 \left(1 - \frac{r_+^{d-1}}{r^{d-1}} \right), \quad (2.2)$$

where we have chosen units such that the AdS radius is unity, and r_+ is radius of the event horizon. The Hawking temperature of the black hole is

$$T = \frac{(d-1)r_+}{4\pi}. \quad (2.3)$$

We now consider the Power-Maxwell field and the charged scalar field coupled via a Lagrangian

$$S = \int d^d x \sqrt{-g} \left[-\beta (F_{\mu\nu} F^{\mu\nu})^q - \partial_\mu \tilde{\psi} \partial^\mu \tilde{\psi} - m^2 \tilde{\psi}^2 - \tilde{\psi}^2 (\partial_\mu p - A_\mu)(\partial^\mu p - A^\mu) \right], \quad (2.4)$$

where $F^{\mu\nu}$ is the strength of the Power-Maxwell field $F = dA$ and $\tilde{\psi}$ is the complex scalar field, and β and q are the coupling constant and the power parameter of the Power-Maxwell field, respectively. The Power-Maxwell field will reduce to the Maxwell case when both $\beta = 1/4$ and $q = 1$. We can use the gauge freedom to fix $p = 0$ and take $\psi \equiv \tilde{\psi}$, $A_t = \phi$ where ψ , ϕ are both real functions of r only. Then the equations of motion are given by

$$\psi'' + \left(\frac{f'}{f} + \frac{d-2}{r} \right) \psi' + \frac{\phi^2}{f^2} \psi - \frac{m^2}{f} \psi = 0, \quad (2.5)$$

$$\phi'' + \left(\frac{d-2}{2q-1} \right) \frac{\phi'}{r} - \frac{2}{(-2)^{1+q} \beta q (2q-1)} \frac{\psi^2 \phi}{f} (\phi')^{2(1-q)} = 0, \quad (2.6)$$

where a prime denotes the derivative with respect to r . At the event horizon $r = r_+$, we must have¹

$$\begin{aligned} \phi(r_+) &= 0, \\ \psi(r_+) &= \frac{(d-1)r_+}{m^2} \psi'(r_+), \end{aligned} \quad (2.7)$$

¹From Eq. (2.6) we know that at the event horizon there may be two cases, i.e., $\phi(r_+) = 0$ or $\phi'(r_+) = 0$. The case of $\phi'(r_+) = 0$ shows that $\phi(r_+) = \text{constant}$. However, the constant must be set to zero according to Gubser's argument in Ref. [5].

and at the asymptotic AdS region ($r \rightarrow \infty$), the solutions behave like (For behavior of ϕ please see Appendix for detail)

$$\begin{aligned}\psi &= \frac{\psi_-}{r^{\lambda_-}} + \frac{\psi_+}{r^{\lambda_+}}, \\ \phi &= \mu - \frac{\rho^{\frac{1}{2q-1}}}{r^{\frac{d-2}{2q-1}-1}},\end{aligned}\tag{2.8}$$

with

$$\lambda_{\pm} = \frac{1}{2} \left[(d-1) \pm \sqrt{(d-1)^2 + 4m^2} \right], \tag{2.9}$$

where μ and ρ are interpreted as the chemical potential and charge density in the dual field theory, respectively. The coefficients ψ_+ and ψ_- both multiply normalizable modes of the scalar field equations and they correspond to the vacuum expectation values $\psi_+ = \langle \mathcal{O}_+ \rangle$, $\psi_- = \langle \mathcal{O}_- \rangle$ of an operator \mathcal{O} dual to the scalar field according to the AdS/CFT correspondence. We can impose boundary conditions that either ψ_+ or ψ_- vanishes. It is of interest to note that the electric field ϕ is dependent on the coupling constant q of the Power-Maxwell field at the asymptotic AdS region, which is different from the Born-Infeld electrodynamics [38] and all known cases.

3. Relations between critical temperature and charge density

In this section we will use both the Sturm-Liouville analytical [39] and numerical methods to calculate the relation between the critical temperature and the charge density of the holographic superconductors in Schwarzschild AdS black hole with the Power-Maxwell field.

Introducing a new coordinate $z = \frac{r_+}{r}$, we can rewrite Eqs. (2.5) and (2.6) as

$$\psi'' + \left(\frac{f'}{f} - \frac{d-4}{z} \right) \psi' + \frac{r_+^2}{z^4} \left(\frac{\phi^2}{f^2} - \frac{m^2}{f} \right) \psi = 0, \tag{3.1}$$

$$\phi'' - \frac{1}{z} \left(\frac{d-2}{2q-1} - 2 \right) \phi' - \frac{2r_+^{2q}}{(-1)^{1+3q} 2^{1+q} \beta q (2q-1) z^{4q}} \frac{\psi^2 \phi}{f} (\phi')^{2(1-q)} = 0, \tag{3.2}$$

here and hereafter a prime denotes the derivative with respect to z . When the temperature T approaches the critical temperature T_c , the condensation approaches zero, viz. $\psi \rightarrow 0$. Thus, Eq. (3.2) becomes

$$\phi'' - \frac{1}{z} \left(\frac{d-2}{2q-1} - 2 \right) \phi' \approx 0. \tag{3.3}$$

The general solution of this equation takes form $\phi = \tilde{a} + \tilde{b} z^{\frac{d-2}{2q-1}-1}$. With help of the boundary conditions (2.7) and (2.8), we find that, near the critical temperature, the electric field can be expressed as

$$\phi = \xi r_+ \left(1 - z^{\frac{d-2}{2q-1}-1} \right), \tag{3.4}$$

where $\xi = \left(\frac{\rho}{r_+^{d-2}}\right)^{\frac{1}{2q-1}}$.

Near the event horizon, we introduce a trial function $F(z)$ into ψ as in Ref. [39]

$$\psi|_{z=0} \sim \frac{\psi_i}{r_+^{\lambda_i}} \sim \langle \mathcal{O}_i \rangle \frac{z^{\lambda_i}}{r_+^{\lambda_i}} F(z), \quad (3.5)$$

here and hereafter subscript $i = (+, -)$. The trial function should satisfy $F(0) = 1$ and $F'(0) = 0$. Then, using Eqs. (3.4) and (3.5), the equation (3.1) for ψ can be rewritten as

$$\begin{aligned} F''(z) &+ \left\{ \frac{2\lambda_i}{z} - \left[\frac{2 + (d-3)z^{d-1}}{z(1-z^{d-1})} + \frac{d-4}{z} \right] \right\} F'(z) \\ &+ \left\{ \frac{\lambda_i(\lambda_i-1)}{z^2} - \frac{\lambda_i}{z} \left[\frac{2 + (d-3)z^{d-1}}{z(1-z^{d-1})} + \frac{d-4}{z} \right] \right. \\ &\left. + \frac{1}{1-z^{d-1}} \left[\frac{\xi^2(1-z^{\frac{d-2}{2q-1}-1})^2}{1-z^{d-1}} - \frac{m^2}{z^2} \right] \right\} F(z) = 0. \end{aligned} \quad (3.6)$$

Multiplying the above equation with the following functional

$$T(z) = z^{2\lambda_i-d+2}(z^{d-1} - 1), \quad (3.7)$$

we can express Eq. (3.6) as

$$[T(z)F'(z)]' - Q(z)F(z) + \xi^2 P(z)F(z) = 0, \quad (3.8)$$

with

$$\begin{aligned} Q(z) &= -T(z) \left\{ \frac{\lambda_i(\lambda_i-1)}{z^2} - \frac{\lambda_i}{z} \left[\frac{2 + (d-3)z^{d-1}}{z(1-z^{d-1})} + \frac{d-4}{z} \right] - \frac{m^2}{z^2(1-z^{d-1})} \right\}, \\ P(z) &= T(z) \frac{(1-z^{\frac{d-2}{2q-1}-1})^2}{(1-z^{d-1})^2}. \end{aligned} \quad (3.9)$$

By using Sturm-Liouville method [39] to solve the Eq. (3.8), we know that the minimum of eigenvalues of ξ^2 can be obtained from the variation of the following expression

$$\xi^2 = \frac{\int_0^1 T(z)F'(z)^2 dz + \int_0^1 Q(z)F(z)^2 dz}{\int_0^1 P(z)F(z)^2 dz}. \quad (3.10)$$

The trial function $F(z)$ can be taken as $F(z) = 1 - az^2$ which satisfies its boundary condition. Then ξ^2 can be explicitly written as

$$\xi^2(d, q, m, a) = \frac{s(d, q, m, a)}{t(d, q, m, a)}. \quad (3.11)$$

For different values of d , q and m , we can find the minimum value of ξ^2 with appropriate value of a . For example, taking $d = 4$ and $q = 3/4$, we have

$$\begin{aligned}
s(4, 3/4, m, a) &= -\frac{12a^2}{4m^2 + 11\sqrt{4m^2 + 9} + 37} - \frac{1}{2} \left(2m^2 + 3\sqrt{4m^2 + 9} + 9 \right) \\
&\quad \times \left(\frac{a^2}{\sqrt{4m^2 + 9} + 7} - \frac{2a}{\sqrt{4m^2 + 9} + 5} + \frac{1}{\sqrt{4m^2 + 9} + 3} \right), \\
t(4, 3/4, m, a) &= -\frac{3a^2}{4m^2 + 15\sqrt{4m^2 + 9} + 63} + \frac{6a}{4m^2 + 11\sqrt{4m^2 + 9} + 37} \\
&\quad - \frac{3}{4m^2 + 7\sqrt{4m^2 + 9} + 19}.
\end{aligned} \tag{3.12}$$

From which we can obtain $\xi_{min} = 3.96555$ with $m^2 = 0$ when $a = 0.71075$, $\xi_{min} = 3.30506$ with $m^2 = -1$ when $a = 0.63430$, $\xi_{min} = 2.28183$ with $m^2 = -2$ when $a = 0.47874$, and $\xi_{min} = 1.5073$ with $m^2 = -9/4$ when $a = 0.33117$.

With the help of $T = \frac{(d-1)r_+}{4\pi}$ and $\xi = \left(\frac{\rho}{r_+^{(d-2)}} \right)^{\frac{1}{2q-1}}$, we know that, when $T \sim T_c$, the critical temperature T_c can be expressed as

$$T_c = \gamma \rho^{\frac{1}{d-2}}, \tag{3.13}$$

where the coefficient $\gamma = \frac{d-1}{4\pi \xi_{min}^{(2q-1)/(d-2)}}$.

In Tables 1 and 2, we list the analytical values and numerical values of critical temperature for different q and m in the 4-dimensional and 5-dimensional black holes, respectively. Some numerical data for critical temperature with $q = 1$ are taken from the Ref. [8]. The differences between the analytical and numerical values are within 5%.

m^2	$q = \frac{3}{4}$				$q = 1$			
	T_c for λ_+		T_c for λ_-		T_c for λ_+		T_c for λ_-	
	Analytical	Numerical	Analytical	Numerical	Analytical	Numerical	Analytical	Numerical
0	$0.1692\rho^{\frac{1}{2}}$	$0.1694\rho^{\frac{1}{2}}$	—	—	$0.0844\rho^{\frac{1}{2}}$	$0.0870\rho^{\frac{1}{2}}$	—	—
-2	$0.1942\rho^{\frac{1}{2}}$	$0.1943\rho^{\frac{1}{2}}$	$0.2529\rho^{\frac{1}{2}}$	$0.2528\rho^{\frac{1}{2}}$	$0.1170\rho^{\frac{1}{2}}$	$0.1180\rho^{\frac{1}{2}}$	$0.2250\rho^{\frac{1}{2}}$	$0.2260\rho^{\frac{1}{2}}$
$-\frac{9}{4}$	$0.2155\rho^{\frac{1}{2}}$	$0.2154\rho^{\frac{1}{2}}$	$0.2155\rho^{\frac{1}{2}}$	$0.2154\rho^{\frac{1}{2}}$	$0.1507\rho^{\frac{1}{2}}$	$0.1520\rho^{\frac{1}{2}}$	$0.1507\rho^{\frac{1}{2}}$	$0.1520\rho^{\frac{1}{2}}$

Table 1: The critical values of T_c for different q and m in 4-dimensional black hole. The numerical data for critical temperature with $q = 1$ are taken from the Ref. [8]

From tables I and II, we find that, for the same q , the critical temperature for the scalar operators $\langle \mathcal{O}_+ \rangle$ decreases as the value of m^2 increases, which means that the larger mass (m^2 becomes less negative) of the scalar field makes it harder for the scalar hair to be condensated in both 4-dimensional and 5-dimensional Schwarzschild AdS black holes, which agrees with the finding in Ref. [8].

From the tables we also know that, for the same mass m and fixed scalar operators $\langle \mathcal{O}_i \rangle$ with $i = (+, -)$, the ratio $T_c/\rho^{1/(d-2)}$ decreases as the q increases, which

m^2	$q = \frac{3}{4}$		$q = 1$		$q = \frac{5}{4}$	
	T_c for λ_+		T_c for λ_+		T_c for λ_+	
	Analytical	Numerical	Analytical	Numerical	Analytical	Numerical
0	$0.2503\rho^{\frac{1}{3}}$	$0.2505\rho^{\frac{1}{3}}$	$0.1676\rho^{\frac{1}{3}}$	$0.1700\rho^{\frac{1}{3}}$	$0.0954\rho^{\frac{1}{3}}$	$0.1008\rho^{\frac{1}{3}}$
-1	$0.2543\rho^{\frac{1}{3}}$	$0.2545\rho^{\frac{1}{3}}$	$0.1739\rho^{\frac{1}{3}}$	$0.1765\rho^{\frac{1}{3}}$	$0.1014\rho^{\frac{1}{3}}$	$0.1065\rho^{\frac{1}{3}}$
-2	$0.2596\rho^{\frac{1}{3}}$	$0.2746\rho^{\frac{1}{3}}$	$0.1825\rho^{\frac{1}{3}}$	$0.1847\rho^{\frac{1}{3}}$	$0.1099\rho^{\frac{1}{3}}$	$0.1145\rho^{\frac{1}{3}}$
-3	$0.2677\rho^{\frac{1}{3}}$	$0.2642\rho^{\frac{1}{3}}$	$0.1962\rho^{\frac{1}{3}}$	$0.1980\rho^{\frac{1}{3}}$	$0.1240\rho^{\frac{1}{3}}$	$0.1279\rho^{\frac{1}{3}}$

Table 2: The critical values of T_c for different q and m in 5-dimensional black hole. The some numerical data for critical temperature with $q = 1$ are taken from the Ref. [8]

means that the larger power q for the Power-Maxwell field makes it harder for the scalar hair to be condensated in the Schwarzschild AdS black hole.

4. Critical exponents

In this section, we will study the critical exponents of the holographic superconductor model with the Power-Maxwell field by using analytical and numerical methods, respectively.

Now we are in position to investigate the critical exponents analytically. From last section we know that the condensation value of the dual operator $\langle \mathcal{O}_i \rangle$ is very small when $T \rightarrow T_c$. Substituting Eq. (3.5) into Eq. (3.2), we have

$$\phi'' - \frac{1}{z} \left(\frac{d-2}{2q-1} - 2 \right) \phi' = \frac{2r_+^{2q-2\lambda_i-2} \langle \mathcal{O}_i \rangle^2}{(-1)^{1+3q} 2^{1+q} \beta q (2q-1)} \frac{z^{2\lambda_i-4q} F^2(z) \phi}{g(z)} (\phi')^{2(1-q)}, \quad (4.1)$$

where $g(z) = (1 - z^{d-1})/z^2$. Note that, near the critical temperature, Eq. (3.4) can be rewritten as

$$\phi = \frac{AT_c^{\frac{d-2}{2q-1}}}{T^{\frac{d-2}{2q-1}-1}} \left(1 - z^{\frac{d-2}{2q-1}-1} \right), \quad (4.2)$$

where $A = \frac{d-1}{4\pi}$. Therefore, we can assume that the general solution for Eq. (4.1) takes the form

$$\phi = AT_c (1 - z^{\frac{d-2}{2q-1}-1}) + (AT_c)^m \left[\frac{r_+^{2q-2\lambda_i-2} \langle \mathcal{O}_i \rangle^2}{(-1)^{1+3q} 2^{1+q} \beta q (2q-1)} \right]^n \chi(z), \quad (4.3)$$

with

$$n = 1, \quad m = 3 - 2q. \quad (4.4)$$

Thus, Eq. (4.1) becomes

$$\chi''(z) - \frac{1}{z} \left(\frac{d-2}{2q-1} - 2 \right) \chi'(z) = \frac{z^{2[\lambda_i-q-1+(\frac{d-2}{2q-1}-1)(1-q)]} \left(1 - z^{\frac{d-2}{2q-1}-1} \right) F^2(z)}{g(z)}, \quad (4.5)$$

which shows us that the functional $\chi(z)$ is independent on r_+ , T_c and $\langle \mathcal{O}_i \rangle$. For examples: if we take $d = 5$ and $q = 3/4$, from Eq. (4.5) we can easily find that $\chi(z) = c_1 + c_2 z^5 + \left[-\frac{a^2 z^8}{4\lambda_i^2 + 22\lambda_i + 24} + a \left(\frac{z}{2\lambda_i^2 + 7\lambda_i + 3} + \frac{1}{2\lambda_i^2 + 5\lambda_i} \right) z^5 + \frac{z^3}{-4\lambda_i^2 - 2\lambda_i + 6} \right] z^{2\lambda_i}$; and if we take $d = 5$ and $q = 1$, we have $\chi(z) = c_1 + c_2 z^2 + \frac{1}{4\lambda_i(\lambda_i^2 - 1)} \left[-(\lambda_i + 1) z^{2\lambda_i} + (2a + 1)(\lambda_i - 1) z^{2\lambda_i + 2} - (a + 1)^2 (\lambda_i - 1) \lambda_i \Gamma(\lambda_i + 2) {}_2\tilde{F}_1(\lambda_i + 1, 1; \lambda_i + 3; -z^2) z^{2\lambda_i + 4} \right]$. That is to say, Eq. (4.5) tells us that $\chi(z)|_{z=0} = c_1$ is a constant which is also independent on r_+ , T_c and $\langle \mathcal{O}_i \rangle$.

At the boundary $z = 0$, from Eqs. (4.2) and (4.3) we have

$$A \frac{T_c^{\frac{d-2}{2q-1}}}{T^{\frac{d-2}{2q-1}-1}} - (AT_c) = \left[\frac{r_+^{2q-2\lambda_i-2} \langle \mathcal{O}_i \rangle^2}{(-1)^{1+3q} 2^{1+q} \beta q (2q-1)} \right] (AT_c)^{3-2q} c_1. \quad (4.6)$$

After some calculations, Eq. (4.6) can be casted into

$$\frac{\langle \mathcal{O}_i \rangle}{T_c^{\lambda_i}} = D \left(\frac{T}{T_c} \right)^{\lambda_i + (1-q)} \left\{ \left(\frac{T_c}{T} \right)^{\frac{d-2}{2q-1}-1} \left[1 - \left(\frac{T}{T_c} \right)^{\frac{d-2}{2q-1}-1} \right] \right\}^{\frac{1}{2}}, \quad (4.7)$$

where constant D is independent on $\langle \mathcal{O}_i \rangle$, T and T_c . Note that our result (4.7) is valid for both of the scalar operators $\langle \mathcal{O}_+ \rangle$ and $\langle \mathcal{O}_- \rangle$ with various Power-Maxwell parameters q and mass m of the scalar field. It is interesting to point out that the critical exponent of the system is equal to $1/2$ which is in agreement with the mean field value.

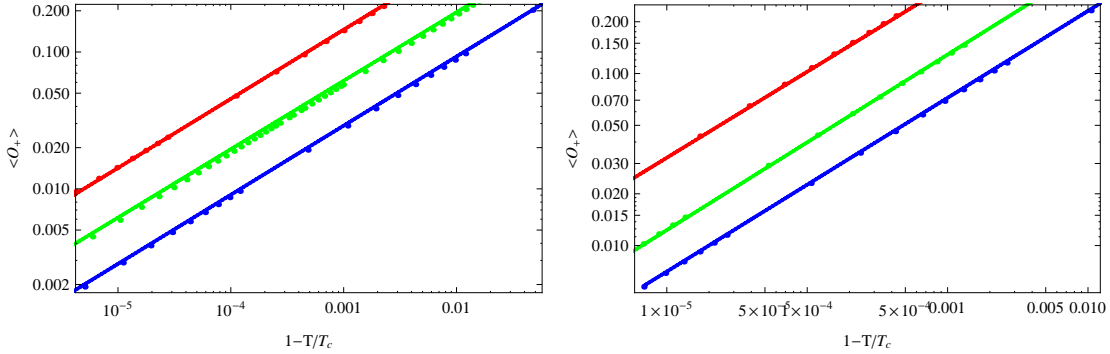


Figure 1: (color online) The condensate $\langle \mathcal{O}_+ \rangle$ vs $1 - T/T_c$ in logarithmic scale with different values of q for $d = 4$ (left) and $d = 5$ (right). The three lines from bottom to top in left panel correspond to $q = 0.75$ (blue), 0.85 (green) and 1.0 (red), and in right one correspond to $q = 0.875$ (blue), 1.0 (green) and 1.25 (red). These panels show that the slope is independent of q .

To check the analytical result (4.7) obtained by using the analytical method, we calculate the critical exponent of the system by using numerical approach. In Fig.

1, we present the condensate $\langle \mathcal{O}_+ \rangle$ as a function of $(1 - T/T_c)$ in logarithmic scale with different values of q for $d = 4$ (left) and $d = 5$ (right). The three lines from bottom to top in left panel correspond to $q = 0.75, 0.85$ and 1.0 , and in right one correspond to $q = 0.875, 1.0$ and 1.25 . We see from these panels that the slope is almost independent of the power parameter q and the mass m of the scalar field, which is in agreement with the analytical value $1/2$.

5. conclusions

The behaviors of the holographic superconductors have been investigated by introducing a complex charged scalar field coupled with a Power-Maxwell field in the background of a planar Schwarzschild AdS black hole. We present a detail analysis of the condensation of the operators by using both the Sturm-Liouville analytical and numerical methods. We first note that the Power-Maxwell field takes the special asymptotical solution near the boundary which is different from all known cases. It is interesting to find that, if we fix the mass parameter m , the critical temperature decreases as the q increases, which means that the larger power parameter q for the Power-Maxwell field makes it harder for the scalar hair to be condensated. For the same q , the critical temperature decreases as the value of m^2 increases, which means that the scalar hair can be formed more difficult for the larger mass of the scalar field in both 4-dimensional and 5-dimensional Schwarzschild AdS black holes, which agrees with the finding in Ref. [8] when $q = 1$. We finally find that, for both the scalar operators $\langle \mathcal{O}_+ \rangle$ and $\langle \mathcal{O}_- \rangle$ with different power parameters q and masses m , the critical exponent of the system is always $1/2$, which seems to be an universal property for various nonlinear electrodynamics if the scalar field ψ takes the form of this paper.

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Appendix

A. ϕ behavior at the asymptotic AdS region

In the AdS/CFT duality [2, 3], it is well known that the bulk gauge field A_μ acts as a source for a conserved current J^μ corresponding to a global U(1) symmetry. In general, near the boundary, the equation of motion for A_μ has the solution

$$A_\mu = a_\mu - \frac{b_\mu}{r^c} + \dots, \quad (\text{A.1})$$

where a_μ and b_μ are constant which physical properties will be discussed in following. Given this expansion, the one-point function of the corresponding field theory operator can be expressed as variation of the action with respect to the boundary value [15]

$$\langle J^\mu \rangle \sim \frac{\delta S}{\delta a_\mu}. \quad (\text{A.2})$$

From the fact that $\langle J^\mu \rangle$ and a_μ are canonically conjugate, we can identify a_0 with the chemical potential μ . Then, the charge density ρ in field theory is defined as [15]

$$\rho = \langle J^0 \rangle. \quad (\text{A.3})$$

For the system that the Power-Maxwell field is coupled with the charged scalar field, we have the solution $A_0 = \phi = a_0 - b_0/r^{\frac{d-2}{2q-1}-1}$ near the boundary. Thus, on shell, i.e. evaluated for a solution to the classical equations of motion, the action reduces to a boundary term of the form

$$S = \left[B a_0 b_0^{2q-1} + (d-1)\Psi_+\Psi_1 + \lambda_- r_B^{d-1-2\lambda_-}\Psi_-^2 \right] \cdot \int d^{d-1}x \sqrt{-g_{d-1}} + \dots, \quad (\text{A.4})$$

where $B = -(-2)^q \beta \left(\frac{d-2}{2q-1} \right)^{2q-1}$. Using Eqs. (A.2), (A.3) and (A.4), we know that the charge density is given by

$$\rho = b_0^{2q-1}. \quad (\text{A.5})$$

Therefore, at the asymptotic AdS region ($r \rightarrow \infty$), the solution behave for function ϕ is

$$\phi = \mu - \frac{\rho^{\frac{1}{2q-1}}}{r^{\frac{d-2}{2q-1}-1}}. \quad (\text{A.6})$$

We should point out that the electric field ϕ is dependent on the coupling constant q at the asymptotic AdS region, which is different from all known cases.

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